Convex MINLP test problems with non-separable nonlinear functions

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August 15, 2017

These test problems are intended to test the capability of solvers to handle convex MINLP problems that are strongly nonlinear, and where the Hessian of the Lagrangian is dense. Solvers based on outer approximation and extended cutting plane method, tend to perform poorly for such problems. This can be explained by the fact that the linear approximations tend to become worse with more non-zero elements in the Hessian. Most of the test problems are thus quite difficult to solve with both BONMIN-OA, AlphaECP and DICOP, even if the largest of these problems only contains 20 continuous and 20 integer variables with ”nice” variable bounds. Some of them are not even solved within 2 hours with the solvers. However, the smaller instances with only 20 variables are more manageable, and most of the smaller instances can be solved within a couple of minutes.

These problems are a bit different than most of the convex MINLP instances available in MINLPLib. For these problems the solvers struggles with handling the nonlinearity and the difficulty is not mainly due to the integer requirements. The MILP sub-problems are solved quickly, but the difficulty is the large number of such subproblems. A nonlinear branch and bound approach such as SBB, is actually much more efficient for these problems than OA.

However, by applying appropriate transformations to the test problems it is possible to make the nonlinear functions separable, and in the separable form it is possible to use a lifted reformulation to obtain tighter linear approximations. The problems where the name contains an R are written is such a lifted form. In the lifted form the problems are quite easy to solve with both BONMIN-OA, AlphaECP and DICOPT, and most are solved within only a few seconds.

Name	*Number of variables*	*Number of integer variables*
N-sig20	*20*	*10*
N-sig30	*30*	*15*
N-sig40	*40*	*20*
P-sig20	*20*	*10*
P-sig30	*30*	*15*
P-sig40	*40*	*20*

Table 1: Number of variables in the signomial test problems.

1 Signomial test problems

The first type of problems contains a convex signomial constraint of the following type

$$\begin{aligned}
&\text{minimize } \mathbf{c}^T\mathbf{x} \\
&\text{subject to} \\
&-a\prod_{i=1}^{N} x_i^{p_i} \leq -1, \\
&0.00001 \leq x_i \leq 10 \quad \forall\, i = 1,\ldots,N, \\
&\mathbf{x} \in \mathbb{R}^N, \\
&x_i \in \mathbb{Z} \quad \forall\, i = 1,\ldots,J,
\end{aligned} \tag{N-sig}$$

where a is a strictly positive constant. The vectors $\mathbf{c}$ and coefficients p_i have been chosen randomly. Furthermore, we consider signomial MINLP problems of the following type

$$\begin{aligned}
&\text{minimize } a\prod_{i=1}^{N} x_i^{p_i} + \sum_{i=1}^{N} x_i \\
&\text{subject to} \\
&1 \leq x_i \leq 10 \quad \forall i = 1,2,...,N, \\
&\mathbf{x} \in \mathbb{R}^N \\
&x_i \in \mathbb{Z} \quad \forall i = 1,\ldots J,
\end{aligned} \tag{P-sig}$$

where $a \in \mathbb{R}_+$ and $p_i \in \mathbb{R}_-$. Here the coefficients p_i are chosen randomly. The size of these test problems are summarized in Table 1.

2 Test problems for the power transform

These are test problems that can be transformed into a separable form by a simple power transform. One such type MINLP problems are

$$\begin{aligned}
&\text{minimize } \mathbf{c}^T \mathbf{x} \\
&\text{subject to} \\
&\|\mathbf{x}\|_2 \le a, \\
&0 \le x_i \le 5 \quad \forall i = 1,2,\dots,N. \\
&\mathbf{x} \in \mathbb{R}^N \\
&x_i \in \mathbb{Z} \quad \forall i = 1,\dots J,
\end{aligned} \tag{Norm-con}$$

where a is a positive constant. Another type of test problems considered are given by

$$\begin{aligned}
&\text{minimize } \mathbf{c}^T \mathbf{x} \\
&\text{subject to} \\
&\left(\sum_{i=1}^{N-1} a^{x_i + x_{i+1}}\right)^p \le b, \\
&0 \le x_i \le 5 \quad \forall i = 1,2,\dots,N. \\
&\mathbf{x} \in \mathbb{R}^N, \\
&x_i \in \mathbb{Z} \quad \forall i = 1,\dots J,
\end{aligned} \tag{P-con}$$

where $a, p > 1$ and b is a constant. The size of these test problems are given in Table 2.

Name	*Number of variables*	*Number of integer variables*
Norm-con20	*20*	*10*
Norm-con30	*30*	*15*
Norm-con40	*40*	*20*
P-con20	*20*	*10*
P-con30	*30*	*15*
P-con40	*40*	*20*

Table 2: Number of variables in the non-separable test problems.